Influence of the Turbulent Schmidt Number on Mass Transfer Rates Between Turbulent Fluid Stream and a Solid Surface

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The objectives of this work are twofold: to examine the effect of turbulent Schmidt number on mass transfer coefficients and to present a new experimental method for determination of mass transfer rates in turbulent flow at large Schmidt numbers.

The correlations for mass transfer coefficients between a turbulent fluid stream and a solid surface can be given by either of the following two forms:

$$Sh = a \, \mathrm{Re} \, f^{b} Sc^{c} \tag{1a}$$

$$Sh = \alpha \, \mathrm{Re}^{\beta} Sc^{c} \tag{1b}$$

Previous investigations did not provide a definite answer concerning the value of the exponents b or β. Some authors use the following representation of the dimensionless mass transfer coefficient:

$$K^{+} = \frac{k}{w\sqrt{\dfrac{f}{2}}} = f(Sc) \tag{2}$$

which, a priori, assumes a value of $b = \frac{1}{2}$. However, Hubbard (1968) has shown that there is no sufficient experimental support for such a postulate. He demonstrated that the experimental accuracy was insufficient to distinguish between the correlation of the form given by Eq. 2 and an alternative dimensionless mass transfer coefficient given by

$$K^{++} = \frac{k}{w\dfrac{f}{2}} \tag{3}$$

Both correlations yield a coefficient that (within experimental accuracy) does not depend on Reynolds number. However, if we use the well-known theoretical relation given by Eq. 4

$$\frac{\epsilon}{\nu} = \gamma (y^{+})^{n} \tag{4}$$

which is valid in the proximity of a solid surface, it would seem at first sight that Eq. 2 has a theoretical foundation, since the value of $b = \frac{1}{2}$ is obtained irrespective of the value of n. However, that is the case only if an additional assumption is made that the turbulent Schmidt number is independent of the Reynolds number. This assumption of constancy of the turbulent Schmidt number has not been proven theoretically nor has it been tested sufficiently by experiment. Here we present some additional data obtained by a new experimental technique to test this relationship further.

In the past, basically two experimental methods have been used for determination of turbulent mass transfer coefficients in liquids. One is the dissolution method (Linton and Sherwood, 1950; Meyerink and Friedlander, 1962; Harriott and Hamilton, 1965) and the other is the electrochemical technique employed by Lin et al. (1951), Van Shaw and Hanratty (1964), Hubbard and Lightfoot (1966), Son and Hanratty (1967), Mizushina et al. (1971), Shaw and Hanratty (1977), and McConaghy and Hanratty (1977). Both methods can be used for determination of average mass transfer coefficients, and the electrochemical technique can also provide local coefficients. Here we introduce a new method based on solute adsorption that provides an alternative to the previous two techniques and might be advantageous to use in certain situations when both local and average coefficients are desired from a single experimental run.

EXPERIMENTAL METHOD AND APPARATUS

In this work, turbulent mass transfer coefficients are obtained by the adsorption method. This method is based on the determination of the amount of the adsorbed solute when the adsorption process is carried under mass transfer controlled conditions. Končar-Djurdjević (1956) has shown that at short adsorption times the rate of adsorption from very dilute solutions depends only on the rate of diffusion through the boundary layer. Silica gel deposited as a thin film on aluminum foils (Merck, DC-Alufolien Kieselgel, Art. 5554) was used as the adsorbent, while methylene blue (Merck, Methylenblau B, Art. 1283) was used as the adsorbing species. In this study, the concentration of methylene blue was $2.5 \cdot 10^{-3}$ g/L, and the entire adsorption period was 5 min. We have already shown (Končar-Djurdjević and Duduković, 1977) that under these conditions the process is mass transfer controlled and that the concentration immediately above the surface of the silica gel is essentially zero.

The amount of adsorbed methylene blue was determined by measuring the reflected light intensity from the exposed foils with silica gel. These foils were set on a flat surface and examined under a microscope with illumination provided by a ray of white light at an angle of 45°. The light is reflected through the lens of the microscope to the photocell where it generates the current, which

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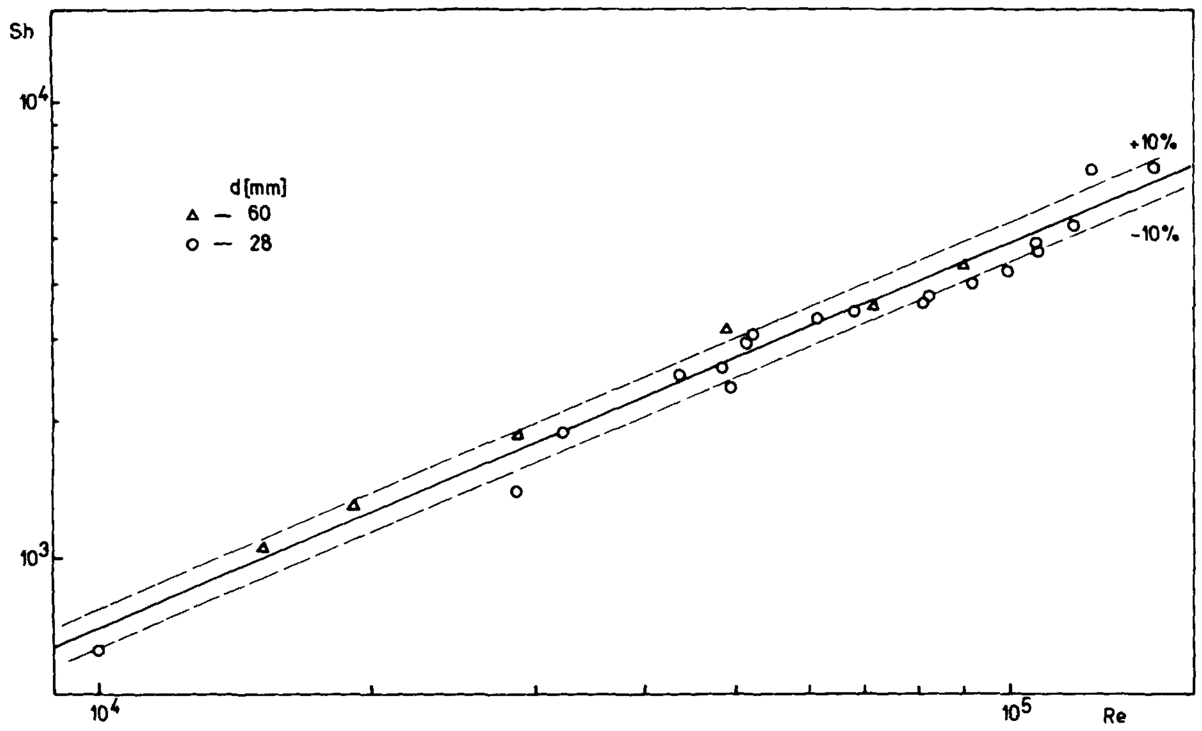

Figure 1. Sherwood number as a function of Reynolds number. Experimental results: ○ 28 mm dia. tube; △ 60 mm dia. tube; Equation 5: ——.

is measured by the multiflex galvanometer (B. Lange). The galvanometer is calibrated to give a reading of 0 for a completely black surface and a reading of 100 for a completely white surface. The microscope is adjusted to have a view area of about 1 mm^2. Therefore the particular value of the mass transfer coefficient obtained is the average value of the local coefficient over this small surface. The relationship between the transferred amount of methylene blue and and reflected light intensity was determined from calibration runs (Cvijović and Mitrović, 1969).

The apparatus consisted of a centrifugal pump and two reservoirs placed on two levels with an overflow on the upper one, which provided constant flow during the course of every experiment. The flow rate was determined by an orifice plate. The pressure drop was measured by U manometers filled with mercury, chloroform, and methylbenzoate. Two test tubes were used with I.D. of 60 and 28 mm. Foils with silica gel were set on the wall of the test tube at the distance of 35 tube diameters from the entrance. This arrangement guaranteed fully developed velocity profiles. The temperature was kept constant at 20°C.

EXPERIMENTAL RESULTS

In measuring the reflections along the foils with silica gel, it was noticed that the change in the intensity of reflected light disappears after a distance of about 1.5 dia. from the leading edge of the silica gel foil. Although the velocity profile and the momentum boundary layer were fully developed at the foils' leading edge, the diffusion boundary layer starts developing at that point and essentially reaches a fully developed thickness after a distance of about 1.5 dia. Hence, this experimental technique is quite useful in studying the length and transport within the developing region. Here mean mass transfer coefficients in the developed region were of interest, and all the coefficients were determined at the distance of 2 dia. from the leading edge, i.e., in the fully developed mass transfer region.

The experiments were carried out for a range of Reynolds numbers from 10^4 to $2 \cdot 10^5$. The results obtained are given in the Figure 1. The circles represent the results for the 28 mm dia. tube, and the triangles are for 60 mm dia. tube. The results are correlated with an equation of the form of Eq. 1b:

$$Sh = 0.032\ Re^{0.842} Sc^{0.296} \tag{5}$$

with the correlation coefficient $r = 0.986$. The value of Schmidt number is $Sc = 1873$. This is based on the diffusion coefficient for methylene blue, which was determined by Končar-Djurdjević (1956) and on the kinematic viscosity of water. The exponent of 0.296 in Eq. 5 has been determined experimentally by Shaw and Hanratty (1977) in systems of various Schmidt numbers. The value of this exponent has no effect on any of the conclusions to be presented. Most of the experimental points lie within ±10% of the correlating line, and all are within ±20%. The results for the friction coefficient are correlated by the equation

$$f = 0.0567\ Re^{-0.222} \tag{6}$$

with the correlation coefficient of $r = 0.997$.

DISCUSSION

The correlations of McConaghy and Hanratty (1977) and Shaw and Hanratty (1977), which contain the assumption of $b = 1/2$, i.e., which use the form of Eq. 2, are compared to our correlation given by Eq. 5 in Figure 2. The discrepancy between our correlation and that of McConaghy and Hanratty (1977) is not larger than the one

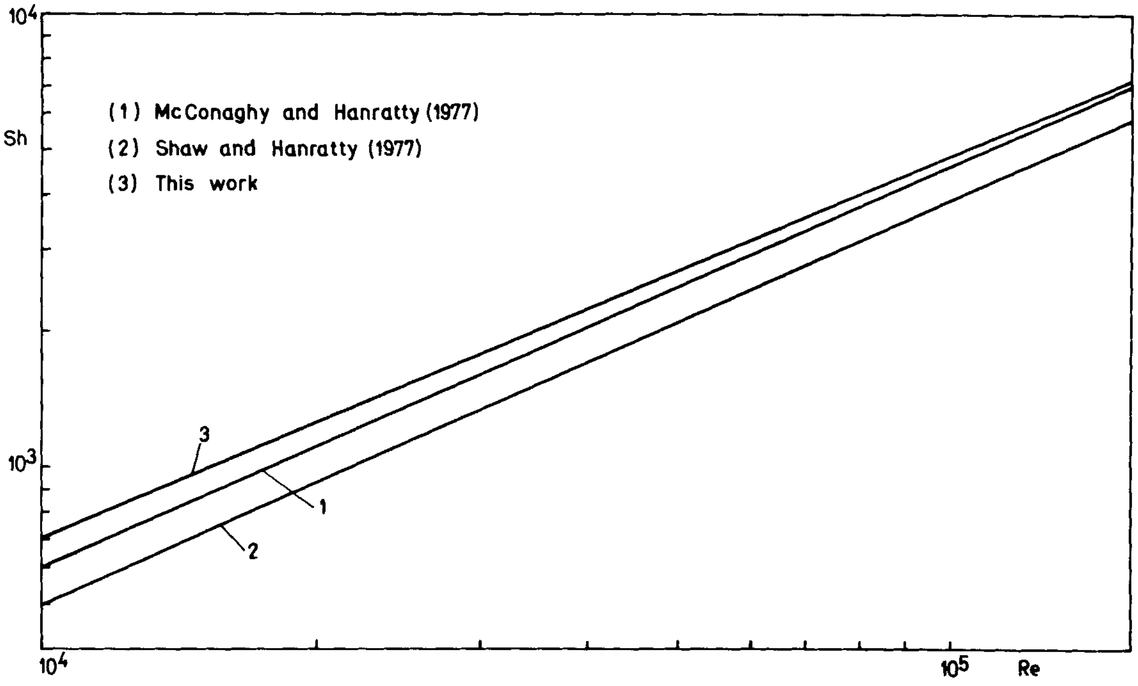

Figure 2. Comparison of various correlations for mass transfer.

between McConaghy and Hanratty (1977) and Shaw and Hanratty (1977). However, the slope of our line is smaller, since we have not assumed the constancy of the turbulent Schmidt number and hence have not taken, a priori, the value $b = 1/2$ as done in the other two correlations. This smaller slope is not the result of errors in one or more experimental points. If one analyzes the results given in Figure 1 it is clear that it would be difficult to find such points that, if discarded, would increase the slope. On the contrary, it would be much easier to do the opposite. On the basis of such considerations, as well as the results of other authors that have obtained b not equal to $1/2$, we come to the conclusion that the smaller slope of the line representing our experiments is not the result of an experimental error.

Equation 2 already contains the assumption that Sc_t = const. Equations 1a and 1b do not contain Sc_t explicitly, but enable an implicit dependence of turbulent Schmidt number on Reynolds number to be reflected in the value of exponent b or β. Our correlations for the mass transfer and friction coefficients give the value $b = 0.712$, which implies the relation

$$Sc_t^{c_1} \sim \mathrm{Re}^{0.047} \tag{7}$$

If it is assumed that $c = c_1$, i.e., that the ratio $(Sc/Sc_t)^c$ is present in Eqs. 1a,b, then one would obtain:

$$Sc_t \sim \mathrm{Re}^{0.159} \tag{8}$$

Equation 8 is limited to the investigated range of Reynolds numbers and to our value of Schmidt number, and it indicates that the turbulent Schmidt number increases with the increase in the Reynolds number. This is the consequence of the shift of the turbulent spectrum toward higher frequency at higher Reynolds numbers and of the fact that these higher-frequency pulsations have a smaller contribution to mass transfer (Sirkar and Hanratty, 1970). Hence both eddy viscosity and eddy mass diffusivity increase with Reynolds number due to the higher turbulence intensity, but the increase of eddy mass diffusivity is slightly slower as the spectrum of frequencies moves toward the higher frequencies.

SUMMARY

The adsorption method presented here can readily be used for experimental determination of local and average mass transfer coefficients in liquids. The data obtained are reproducible and of comparable accuracy to those obtained by other methods.

The correlations for mass transfer, which rely on the assumption of a constant turbulent Schmidt number, such as Eq. 2, are useful due to their simplicity. However, correlations of type 1 might be preferable since they do not rely on that assumption. Our data indicate, although not conclusively, that turbulent Schmidt number varies with Reynolds number. Since there is no theory that proves otherwise, such indications should not be discarded. In a future paper we will present a more compelling argument and proof for the variation of the turbulent Schmidt number.

NOTATION

a,b,c = coefficients in Eqs. 1a,b
f = fanning friction factor
K^+ = local dimensionless mass transfer coefficient defined in Eq. 2
K^{++} = local dimensionless mass transfer coefficient defined in Eq. 3
k = local mass transfer coefficient
n = exponent in Eq. 4
Re = Reynolds number
Sc = Schmidt number
Sc_t = turbulent Schmidt number
Sh = Sherwood number
w = average velocity
y^+ = dimensionless distance from pipe wall

Greek Letters

α,β = coefficients in Eq. 1b
γ = coefficient in Eq. 4
ϵ = eddy viscosity
ν = kinematic viscosity

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Manuscript received Nov. 8, 1983; revision received Sept. 11 and accepted Oct. 30, 1984.